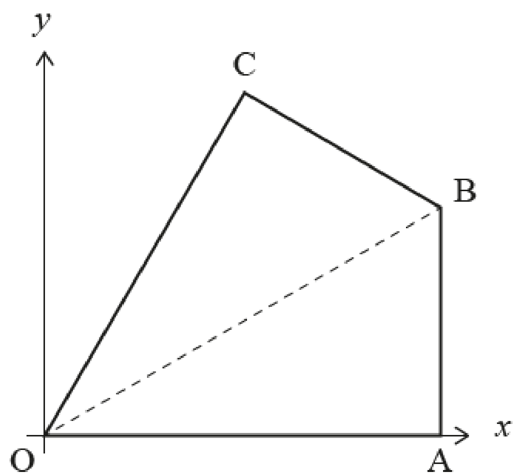
Linear [31 marks]

1. [Maximum mark: 7]

Quadrilateral OABC is shown on the following set of axes.



OABC is symmetrical about $[\mathrm{OB}]$.

A has coordinates $(6,\ 0)$ and C has coordinates $\left(3,\ 3\sqrt{3}\right)$.

(a.i) Write down the coordinates of the midpoint of $[\mathrm{AC}]$. [1]

Markscheme

$\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$ (accept $x = \frac{9}{2}$ and $y = \frac{3\sqrt{3}}{2}$) ***A1***

[1 mark]

(a.ii) Hence or otherwise, find the equation of the line passing through the points O and B. [3]

Markscheme

using $m = \frac{\text{change in } y}{\text{change in } x}$ with their midpoint OR gradient perpendicular to AC

OR $m = \tan 30^\circ$ ***(M1)***

$m = \frac{\sqrt{3}}{3}$ ***(A1)***

$y = \frac{\sqrt{3}}{3}x$ OR $y - \frac{3\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x - \frac{9}{2}\right)$ (must be written as an equation) ***A1***

[3 marks]

(b) Given that $[\mathrm{OA}]$ is perpendicular to $[\mathrm{AB}]$, find the area of the quadrilateral OABC. [3]

Markscheme

substituting $x = 6$ into their equation ***(M1)***

so at B $y = 2\sqrt{3}$ ***(A1)***

area of triangle $\mathrm{OAB} = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$

area of quadrilateral $\mathrm{OABC} = 12\sqrt{3}$ ***A1***

[3 marks]

2. [Maximum mark: 5]

Point P has coordinates $(-3,\ 2)$, and point Q has coordinates $(15,\ -8)$. Point M is the midpoint of $[\mathrm{PQ}]$.

(a) Find the coordinates of M. [2]

Markscheme

$\mathrm{M}\ (6,\ \ -3)$ ***A1A1***

[2 marks]

Line L is perpendicular to $[\mathrm{PQ}]$ and passes through M.

(b) Find the gradient of L. [2]

Markscheme

gradient of $\left[\mathrm{PQ}\right] = -\frac{5}{9}$ ***(A1)***

gradient of $L = \frac{9}{5}$ ***A1***

[2 marks]

(c) Hence, write down the equation of L. [1]

Markscheme

$y + 3 = \frac{9}{5}\left(x - 6\right)$ OR $y = \frac{9}{5}x - \frac{69}{5}$ (or equivalent) ***A1***

Note: Do not accept $L = \frac{9}{5}x - \frac{69}{5}$.

[1 mark]

3. [Maximum mark: 7]

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

(a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. [2]

Markscheme

gradient of g is -2 (may be seen in function, do not accept $-2x + 3$) ***(A1)***

$g(x) = -2x$ ***A1***

[2 marks]

(b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1,\ 2)$. Find an expression for $h(x)$. [3]

Markscheme

gradient is $\frac{1}{2}$ (may be seen in function) ***(A1)***

attempt to substitute **their** gradient and $(-1,\ 2)$ into any form of equation for straight line ***(M1)***

$y - 2 = \frac{1}{2}(x + 1)$ OR $2 = \frac{1}{2} \cdot (-1) + c$

$h(x) = \frac{1}{2}(x + 1) + 2 \ \left(= \frac{1}{2}x + \frac{5}{2}\right)$ ***A1***

[3 marks]

(c) Find $(g \circ h)(0)$. [2]

Markscheme

$(g \circ h)(x) = -2\left(\frac{1}{2}x + \frac{5}{2}\right)$ OR $h(0) = \frac{5}{2}$ OR $g\left(\frac{5}{2}\right)$ ***(A1)***

$(g \circ h)(0) = -5$ ***A1***

[2 marks]

4. [Maximum mark: 5]

Consider the points $\mathrm{A}(-2,\ 20), \mathrm{B}(4,\ 6)$ and $\mathrm{C}(-14,\ 12)$. The line L passes through the point A and is perpendicular to $[\mathrm{BC}]$.

(a) Find the equation of L. [3]

Markscheme

$m_{\mathrm{BC}} = \frac{12-6}{-14-4}\ \left(= -\frac{1}{3}\right)$ ***(A1)***

finding $m_L = \frac{-1}{m_{\mathrm{BC}}}$ using their m_{BC} ***(M1)***

$m_L = 3$

$y - 20 = 3(x+2),\ \ y = 3x + 26$ ***A1***

Note: Do not accept $L = 3x + 26$

[3 marks]

(b) The line L passes through the point $(k,\ 2)$.

Find the value of k. [2]

Markscheme

substituting $(k,\ 2)$ into their L ***(M1)***

$2-20=3(k+2)$ OR $2=3k+26$

$k=-8$ ***A1***

[2 marks]

5. [Maximum mark: 7]

The function f is defined for all $x\in\mathbb{R}$. The line with equation $y=6x-1$ is the tangent to the graph of f at $x=4$.

(a) Write down the value of $f\prime(4)$. [1]

Markscheme

$f\prime(4)=6$ ***A1***

[1 mark]

(b) Find $f(4)$. [1]

Markscheme

$f(4)=6\times4-1=23$ ***A1***

[1 mark]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

(c) Find $h(4)$. [2]

Markscheme

$h(4) = f(g(4))$ ***(M1)***

$h(4) = f\left(4^2 - 3 \times 4\right) = f(4)$

$h(4) = 23$ ***A1***

[2 marks]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

Markscheme

attempt to use chain rule to find $h\prime$ ***(M1)***

$f\prime(g(x)) \times g\prime(x)$ OR $\left(x^2 - 3x\right)\prime \times f\prime\left(x^2 - 3x\right)$

$h\prime(4) = (2 \times 4 - 3) f\prime\left(4^2 - 3 \times 4\right)$ ***A1***

$= 30$

$y - 23 = 30(x - 4)$ OR $y = 30x - 97$ ***A1***

[3 marks]